[3] P. D. Batelaan and M. A. Frerking, "Quantum-well multipliers," *12th Int. Conf. Infrared and Millimeter Waves Dig.* pp. 14–15, 1987.
[4] D. N. Held and A. R. Kerr, "Conversion loss and noise of microwave and millimeter-wave mixers: Pt 1 - Theory," *IEEE Trans. Microwave Theory Tech.*, vol MTT-26, pp. 49–55, 1978.
[5] H. Siegel, A. R. Kerr, and W. Hwang, "Topics in the optimization of MM wave mixers," NASA Tech. Paper 2287, 1987.
[6] R. J. Hwu and N. C. Luhmann, Jr., "Quantum well diode frequency multiplier study," *Second Int. Symp. Space Terahertz Technology,* Proc. pp. 226–237, 1991.
[7] P. P. Huang, "I-V characterization of negative resistance device by microwave reflection coefficients," M.S. thesis, UCLA, pp. 49–51, 1989.

The Introduction of Surface Resistance in the Three-Dimensional Finite-Difference Method in Frequency Domain

Detlev Hollmann, Steffen Haffa, Friedhelm Rostan, and Werner Wiesbeck

***Abstract*—A full-wave treatment of lossy three-dimensional structures using the finite-difference method in frequency domain is presented. This accounts for both, dielectric and conductor losses. By introduction of a surface resistance the effect of conductor losses and surface roughness can be modeled very efficiently. The modifications of the finite-difference frequency domain (FDFD) algorithm are presented. Comparisons between the conventional approach using elementary cells with finite conductivity and this new discretization method with surface current cells are given, and the advantages and limitations of the surface current model are shown.**

I. Introduction

The FDFD method supports an exact description of the discretized electromagnetic field of three-dimensional structures [1]–[3]. Therefore, the method is capable of modeling all phenomena causing attenuation, if elementary cells with a complex permittivity are used. The validity of this method has been shown for the calculation of the complex propagation constant of transmission lines and three-dimensional discontinuity problems [3]. For good conductors with a small skin depth, however, it is necessary to use a very fine discretization at the conductor surfaces in order to achieve the desired accuracy. This increases the computational effort drastically.

To avoid the fine discretization, the paper proposes an infinitely thin current at the conductor surface, which accounts for the physical field penetration. By the use of this technique, a large reduction of the required number of cells needed to model the conductor is achieved.

II. Conventional Modeling of Lossy Structures

The finite difference algorithm has been described for lossless structures [2] and for lossy structures [1] in detail, so only the new developments are presented in this paper.

Assuming a harmonic time dependence Maxwell's equations in integral form (usual definitions) for source free structures are:

$$\oint_C \frac{1}{\mu}\vec{B}\cdot d\vec{s} = \int_A (j\omega\vec{D} + \kappa\vec{E})\cdot d\vec{A} = \int_A j\omega\varepsilon\vec{E}\cdot d\vec{A} \qquad \text{(1a)}$$

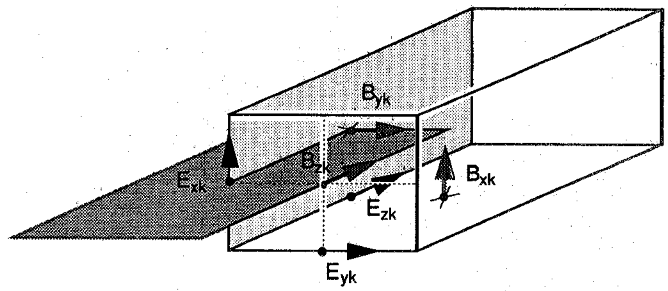

Fig. 1. Elementary cell k and allocation of its associated electric and magnetic field components. The shaded areas indicate integration planes for discretization of Maxwell's equation.

$$\oint_C \vec{E}\cdot d\vec{s} = -\int_A j\omega\vec{B}\cdot d\vec{A} \qquad \text{(1b)}$$

Up to now polarization losses ($\tan\delta$) and the finite conductivity κ have been included in the frequency dependent part of the complex permittivity:

$$\varepsilon = \varepsilon_0\varepsilon_k = \varepsilon_0\left[\varepsilon_r - j\left(\varepsilon_r\tan\delta + \frac{\omega\kappa}{\varepsilon_0}\right)\right] \qquad (2)$$

Maxwell's equations are discretized on a Yee-grid [4]. Fig. 1 shows the elementary cell k and the allocation of the electric and magnetic field components. The shaded areas denote the integration planes for the E_{xk} component ((1a)) and the B_{yk} component ((1b)).

The discretization is performed by a lowest order integration formula. Setting up the associated equations for all E-field and B-field components of the elementary cells for the whole structure leads to a linear system of equations, which defines a boundary value problem for the unknown electric field.

Conductor losses are treated sufficiently when the conductor is discretized with at least three discretization steps per skin depth. Due to the large number of necessary discretizations required inside the conductor the system matrix gets large, and the great differences in the grid spacing influences the convergency behavior of the equation solver negatively. To overcome these drawbacks elementary cells with a surface resistance are introduced.

III. The Improved Method of Analysis

With the new surface resistance approach a non ideal conductor is modeled by its surface resistance. The interior of the conductor is set free of fields and the actual conduction current is represented by an infinitely thin current on the surface of the cells. For that reason the fine discretization inside the conductor becomes no longer necessary. The losses are introduced by a surface resistance R_S, which is inversely proportional to the skin depth δ of the modeled conductor at the radian frequency ω:

$$\delta = \sqrt{\frac{2}{\omega\kappa\mu_0\mu_r}} \qquad (3)$$

Considering the effect of surface roughness σ_{eff} by the correction formula given in [5], [6], leads to an effective surface resistance:

$$R_{S\text{eff}} = R_S\left[1 + \frac{2}{\pi}\arctan\left(1.4\left(\frac{\sigma_{\text{eff}}}{\delta}\right)^2\right)\right] \quad \text{with } R_S = \frac{1}{\kappa\delta} \qquad (4)$$

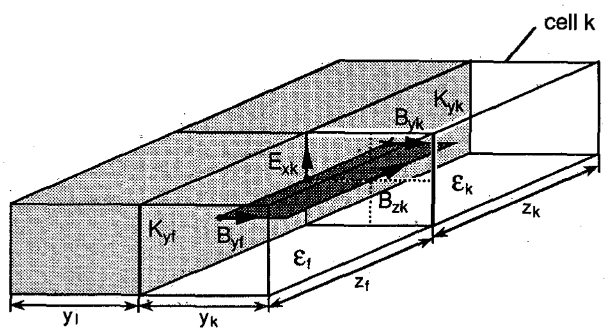

Fig. 2. View of the integration plane for the E_{xk}-component at the surface of a lossy conductor (shaded) showing the allocation of the electric and magnetic field components and the geometrical and material parameters.

with σ_{eff} the root mean square of the conductor surface roughness function $\sigma(x)$.

The introduction of a surface resistance R_S is valid for conductor thicknesses t much greater than the skin depth δ. The approximation becomes more accurate as the skin depth decreases with frequency, or for higher conductivity.

By introducing the surface admittance vector of the cell k

$$\vec{K} = K_{xk}\vec{n}_x + K_{yk}\vec{n}_y + K_{zk}\vec{n}_z$$

with

$$K_i = \frac{1}{R_{S\text{eff},i}} \qquad i = xk, yk, zk \tag{5}$$

the first Maxwell equation is modified at conductors to:

$$\oint_C \frac{1}{\mu}\vec{B}\cdot d\vec{s} = \int_A j\omega\varepsilon\vec{E}\cdot d\vec{A} + \int_l (\vec{E}\times\vec{K})\cdot \vec{d} \tag{6}$$

where the complex dielectric constant ε now accounts for dielectric losses only. The second integral on the right hand side of (6) represents the current on the conductor surface at the edge of the conductor cells, with the integration path along the surface, whereas the first integral accounts for the dielectric displacement current in the neighbouring conventional cells.

Fig. 2 shows the integration plane for the E_{xk}-component at the surface of a lossy conductor. The actual cell k and the neighbouring cell f in front of it are assumed to be conventional cells with relative permittivities ε_k and ε_f. However the shaded cells are denoted to be a lossy conductor characterized by the surface admittances K_{yk} and K_{yf}. The integration in (6) for the E_{xk} -component, Fig. 2, yields:

$$\frac{1}{2}\left(\frac{z_f}{\mu_f}+\frac{z_k}{\mu_k}\right)B_{zk} - \frac{1}{2}\frac{y_k}{\mu_k}B_{yk} + \frac{1}{2}\frac{y_k}{\mu_f}B_{yf} = \frac{j\omega\varepsilon_0\mu_0}{4}(y_k z_k \varepsilon_k + z_f y_k \varepsilon_f) \cdot E_{xk} + \frac{\mu_0}{2}(K_{yf}z_f + K_{yk}z_k)E_{xk} \tag{7}$$

The second of Maxwell's equations is unchanged. Using the discretization scheme described in [1] only the parts resulting from the conductor integral of (6) are added to the discretisized equations, hence the algorithm remains unchanged.

IV. Verification and Results

The improved conductor approach, which utilizes the new surface resistance model, has been implemented in the three-dimensional finite difference method, allowing an accurate treatment of conductor losses and consideration of surface roughness. In order to validate this new technique, the propagation constants of a shielded 50 Ω microstrip line were calculated with the new surface resistance approach and compared to the results obtained by the conventional approach [1] and the mode matching method [7]. Fig. 3 shows the excellent agreement of all three methods. The propagation constants computed by the surface resistance approach converge with the results obtained by the other two methods as the frequency increases. This means that a low frequency limit for this approach exists as is also evident. For good accuracy the smallest conductor thickness should approximately be greater than ten times the skin depth at the lowest frequency.

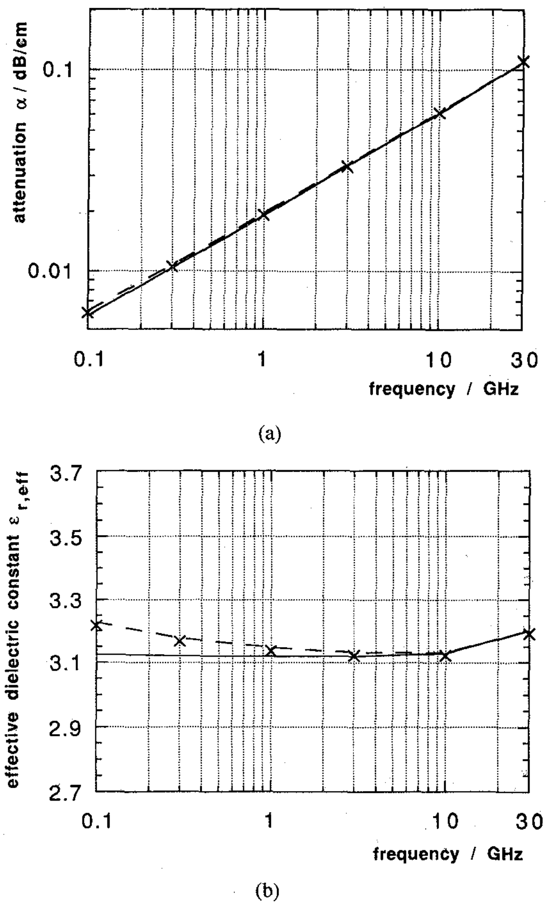

Fig. 3. Comparison of attenuation α (a) and effective relative dielectric constant $\varepsilon_{r,\text{eff}}$ (b) of a shielded 50 Ω microstrip line calculated by a conventional approach [1] (– – –), surface resistance approach (___) and mode matching technique [7] (×). Dimensions of substrate and microstrip: $w = 320\,\mu\text{m}, h = 200\,\mu\text{m}, t = 35\,\mu\text{m}, \kappa = 58\cdot 10^6\,[\Omega\text{m}]^{-1}$, and $\varepsilon_r = 5$. Metallic shielding: $635 \times 900\,\mu\text{m}^2$.

The new method has the advantage of being considerably faster than are conventional methods (8 discretizations in the conductor in contrast to 90 discretizations) and using a frequency independent grid. This is in contrast to the conventional method, which required a variation of grid spacing for different frequencies.

For two problems results are presented. The introduction of a surface resistance in the finite difference method allows the investigation of the effects of surface roughness on the line attenuation. The influence of the roughness at the interfaces between both stripline and substrate and ground metallization and substrate on the attenuation constant is given in Fig. 4. The results agree with the values estimated by the incremental inductance rule.

As an example of a three-dimensional structure, the scattering parameters of a microstrip parallel resonator have been calculated. Fig. 5 shows the S_{12} of the resonator in the case of an ideal conductor

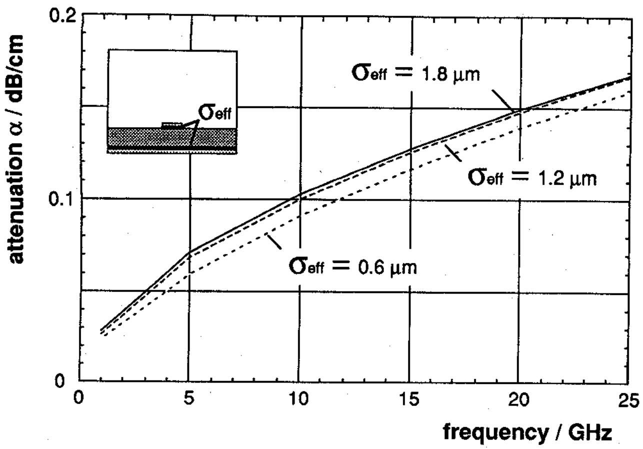

Fig. 4. Total attenuation of a shielded 75 Ω microstrip line versus frequency with surface roughness parameter of σ_{eff} at the interfaces between conductor and substrate. Dimensions of substrate and microstrip: $w = 130\,\mu\text{m}, h = 200\,\mu\text{m}, t = 35\,\mu\text{m}, \kappa = 58 \cdot 10^6 [\Omega\text{m}]^{-1}$, and $\varepsilon_r = 5$. Metallic shielding: $1235 \times 1640\,\mu\text{m}^2$.

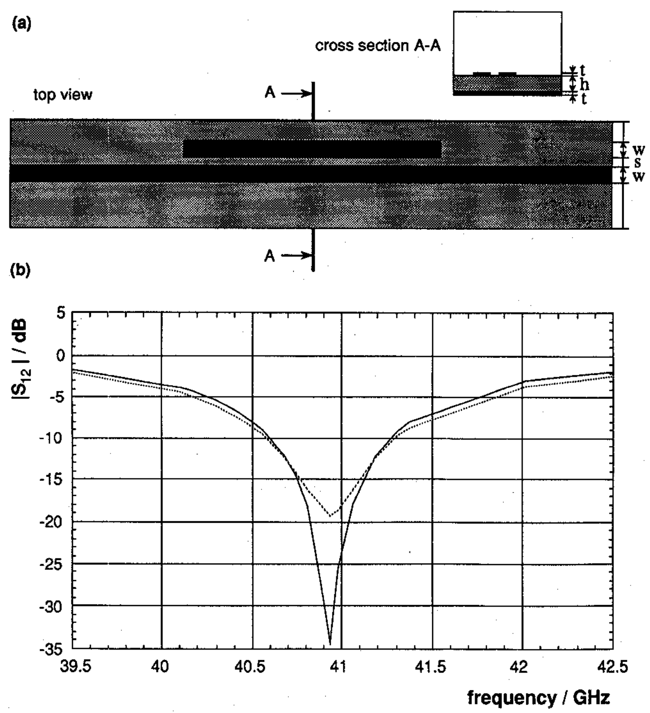

Fig. 5. Configuration (a), and magnitude of the transmission coefficient S_{12} (b), for a microstrip parallel resonator (one wavelength long at resonance), showing the influence of copper metallization and conductor roughness. $w = 150\,\mu\text{m}, h = 150\,\mu\text{m}, t = 30\,\mu\text{m}, s = 75\,\mu\text{m}, \kappa = 58 \cdot 10^6 [\Omega\text{m}]^{-1}$ and $\varepsilon_r = 12.9$. ············ copper $\sigma_{\text{eff}} = 0.25\,\mu\text{m}$. ____ ideal conductor.

compared to a real copper conductor with a finite surface roughness. The resonator quality factor is lowered as expected by the conductor losses.

V. Conclusions

An extension of the finite difference method in frequency domain for accurate and effective treatment of conductor losses and surface roughness was presented. The results confirm that the approach is applicable as long as the conditions for the skin depth are fulfilled. The introduction of discretization cells with a surface resistance reduces the total number of required cells by use of a frequency independent coarse grid. The method has the advantage of giving only a small decrease of convergence stability compared to the lossless calculation.

The proposed method is well suited for CAD.

Acknowledgment

The authors would like to thank Dr. Heinrich of the Ferdinand and Braun Institute Berlin for performing reference calculations with the mode matching method to check the presented method versus his results.

References

[1] S. Haffa, D. Hollmann, and W. Wiesbeck, "The finite difference method for S-parameter calculation of arbitrary three-dimensional structures," *IEEE Trans. Microwave Theory Tech.,* vol. 40, pp. 1602–1610, Aug. 1992.

[2] A. Christ and H. Hartnagel, "Three-dimensional finite-difference method for the analysis of microwave-device embedding," *IEEE Trans. Microwave Theory Tech.,* vol. MTT-35, pp. 688–696, Aug. 1987.

[3] D. Hollmann, S. Haffa, and W. Wiesbeck, "Full wave analysis for S-parameter calculation of arbitrary three-dimensional lossy structures," in *Proc. 12th European Microwave Conf.,* Budapest, Sept. 1990, pp. 1065–1070.

[4] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equation in isotropic media," *IEEE Trans. Antennas Propagat.,* vol. AP-14, pp. 302–307, May 1966.

[5] S. P. Morgan, "Effect of surface roughness on eddy current losses at microwave frequencies," *J. Appl. Phys.,* vol. 20, pp. 352–362, 1949.

[6] R. K. Hoffmann, *Integrierte Mikrowellenschaltungen.* Berlin, Heidelberg, New York: Springer Verlag, 1983, pp. 201–203.

[7] W. Heinrich, "Full-wave analysis of conductor losses on MMIC transmission lines," *IEEE Trans. Microwave Theory Tech.,* vol. 38, pp. 1468–1472, Oct. 1990.

TM-Scattering from a Slit in a Thick Conducting Screen: Revisited

Soo H. Kang, Hyo J. Eom, and Tah J. Park

***Abstract*—TM plane-wave scattering from a slit in a thick conducting screen is reexamined. A Fourier transform technique is employed to express the scattered field in the spectral domain, and the boundary conditions are enforced to obtain simultaneous equations for the transmitted field inside the thick conducting screen. The simultaneous equations are solved to represent the transmitted and scattered fields in series forms. Approximate series solutions for scattering and transmission are obtained in closed-forms which are valid for high-frequency scattering regime.**

Manuscript received April 2, 1992; revised September 10, 1992.

The authors are with the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, 373-1, Kusong Dong, Yusung Gu, Taejon, Korea.

IEEE Log Number 9207434.